II-7. INJECTION-LOCKED OSCILLATORS AS AMPLIFIERS FOR ANGLE-MODULATED SIGNALS

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Introduction. Under certain conditions an oscillator can lock to and track an external driving signal whose frequency is near the free-running frequency of the oscillator and whose amplitude is considerably less than the oscillator output amplitude; this property has led to speculation whether the locked oscillator can serve as an amplifier for angle-modulated signals. We have conducted experimental and theoretical investigations in order to clearly define what the conditions are which would allow such an application. The investigations have included steady-state locking performance, dynamic amplifier response, and noise behavior of the locked oscillators. The experiments were performed on tunnel-diode oscillators at X-band. The theory is valid for a wide number of oscillator types.

Theory of Injection-Locking. A typical negative-resistance oscillator, whose free-running frequency and voltage amplitude are ω_o and A_o, will have an output voltage

$$v(t) \approx A_o \sin (pt-\varphi(t)) \tag{1}$$

when driven by an external signal $E_o a(t) \cos (pt+\theta(t))$ provided the following conditions are met: $|E_o a(t)|_{max} << A_o$; $|p+\dot{\theta}-\omega_o|_{max} \ll \omega_o$; $\theta(t)$ and $a(t)$ are slowly varying functions of time. The slowly varying function $\varphi(t)$ behaves according to the differential equation

$$\dot{\varphi}(t) = (p-\omega_o) - \Delta_o a(t) \sin [\varphi(t)+\theta(t)], \tag{2}$$

where Δ_o is the maximum frequency difference, $|p-\omega_o|$ max, at which locking can occur when the input signal is unmodulated. When the oscillator is locked to an unmodulated signal, the steady-state phase angle φ_{ss} is given by

$$\sin \varphi_{ss} = \frac{(p-\omega_o)}{\Delta_o} . \tag{3}$$

Theory predicts that the locking bandwidth $2\Delta_o/\omega_o$ is linearly proportional to the amplitude ratio E_o/A_o. It is found useful to define a locking figure of merit η in terms of the "voltage-gain x bandwidth" product:

$$\eta = \frac{2\Delta_o}{\omega_o} \times \frac{A_o}{E_o} \approx \text{constant}, \tag{4}$$

where $G \equiv A_0^2/E_0^2$ is the power gain of the amplifier. An expression relating η to circuit parameters may be derived from an assumed model, or η may be determined experimentally. A reflection or attenuation of input signal serves to reduce η.

Under the assumptions made in the theory, the output of the locked oscillator contains only FM sidebands and FM noise. The frequency response of the locked oscillator to low index FM signal sidebands at frequencies $p \pm \omega_1$ is given by

$$\frac{P_{1_{out}}}{P_{1_{in,FM}}} = \frac{G}{1 + \left(\frac{\omega_1}{\Delta_o \cos \varphi_{ss}}\right)^2} ; \qquad (5)$$

the frequency response is thus that of a first order Butterworth (maximally flat) characteristic. When broadband white noise is admitted at the input along with the locking signal, the oscillator output noise spectrum is described by

$$\frac{N_{out/cycle}}{N_{in/cycle}} = \frac{1}{2} \sec^2 \varphi_{ss} \frac{G}{1 + \left(\frac{\omega_1}{\Delta_o \cos \varphi_{ss}}\right)^2} \qquad (6)$$

The AM to FM conversion goes as $\tan^2\varphi_{ss}$; since $\sec^2\varphi_{ss} = 1+\tan^2\varphi_{ss}$, the AM to FM noise conversion is easily identified in Eq. 6. To minimize AM to FM conversion of noise, the locking signal should be near the free-running frequency of the oscillator so that $\varphi_{ss} \approx 0$. When driven at $p = \omega_o$, the locked oscillator behaves as a linear amplifier followed by a limiter and a band-pass filter as shown in Fig. 1.

It was found that the two steady-state parameters, Δ_o and φ_{ss}, characterize the locked oscillator for a given system application (for example, various forms of FM, PM, and PCM). Other system parameters are simply related to those two parameters.

Experiment. Fig. 2 shows a block diagram of the experimental apparatus utilized to study the basic steady-state behavior and then the noise performance of the tunnel-diode locked oscillators. The frequency of the locking signal was swept and the locking range of the tunnel-diode oscillator was measured as a function of gain G; a wavemeter and oscilloscope were generally employed for this purpose. The variation of the steady-state phase angle φ_{ss} with Δ_o was determined by holding the signal frequency at some fixed $p \neq \omega_o$, and adjusting the phase shifter and attenuator in Path A so that the output carrier of the locked oscillator was canceled in the detector; the phase shifter setting required to maintain the "null" in the detector was then measured as a function of locked oscillator gain G.

In Fig. 3 are plots of fractional locking bandwidth for several diodes versus gain G. Locking was measured from 7 to 70 db gain. For gains above 30 db, any oscillator output reflected from the external circuit can be the same order as the locking signal, and so points beyond 30 db gain may not fall on a straight line. The slope of the lines agree with theory (Eq. 4). The vertical intercept is equal to η, the locking figure of merit. A 10% locking bandwidth (1.1 Gc) at 20 db gain was obtained for some diodes. Approximately a third of the diodes characterized had values of η greater than 0.5; most had values $\eta \approx 0.3$-0.4. A correlation between locking figure of merit and rf efficiency of the oscillator was noted --the higher the rf efficiency, the greater the locking figure of merit. Fig. 4 compares measurements of the steady-state phase angle φ_{ss} to theory (Eq. 3).

Noise performance of the phase-locked oscillator was investigated by measuring the output noise spectrum when the input locking signal was (1) a cw signal alone and (2) a cw signal plus wideband white noise. The oscillator output carrier was canceled in the converter by the signal from Path A so that only the noise sidebands were detected. The receiver local oscillator was swept, and the noise spectra shown in Fig. 5 were obtained; the upper and lower sidebands of the local oscillator are present. Fig. 5a displays how the bandwidth of the spectrum decreases with gain. In Fig. 5b, enough broadband white noise was added at the input so that the output spectrum was raised 3 db and 12 db respectively; the shape of the spectrum is the same in all cases and therefore indicates that the equivalent noise source in the amplifier is a broadband white noise source.

Fig. 6 compares a typical measured noise spectrum to theory. Theory and experiment agree very well when a noise figure of 6-7 db is assigned to the equivalent noise source shown in Fig. 1. The noise figure was also verified independently by calibrating the input noise necessary to raise the output spectrum 3 db. The 6-7 db noise figure represents both the AM and FM noise contributed by a white noise source; the limiting action of the oscillator can remove the AM noise so that the noise figure for some system applications may be effectively only 3-4 db.

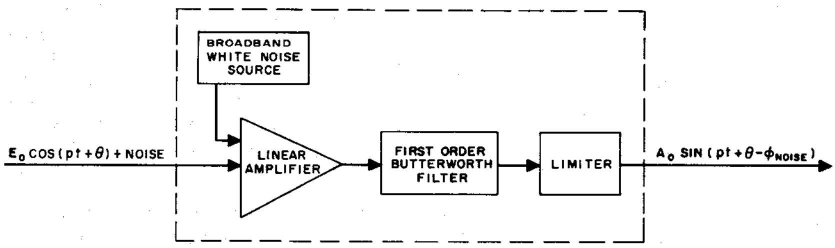

FIGURE 1: EQUIVALENT BLOCK DIAGRAM OF PHASE-LOCKED OSCILLATOR WHEN $\phi_{ss} \approx 0$; i.e., WHEN $(p-\omega_0) << \Delta_0$

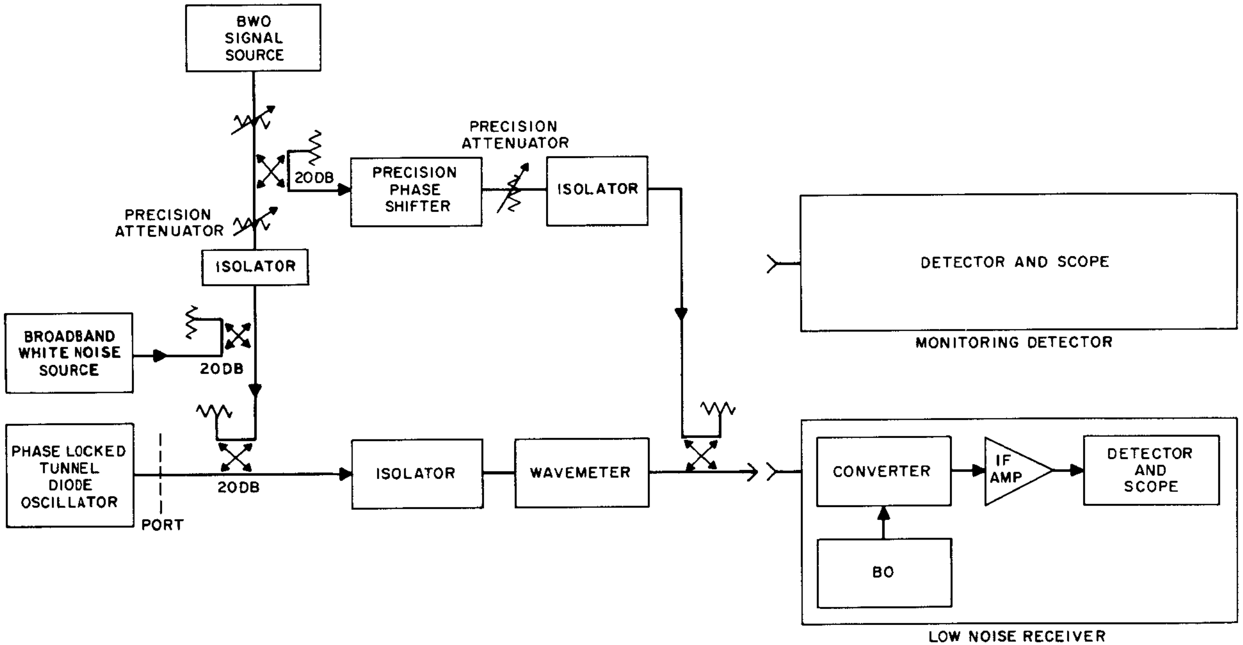

FIGURE 2

EXPERIMENTAL ARRANGEMENT FOR MEASUREMENT OF STEADY-STATE BEHAVIOR AND NOISE PERFORMANCE OF PHASE LOCKED TUNNEL DIODE OSCILLATORS

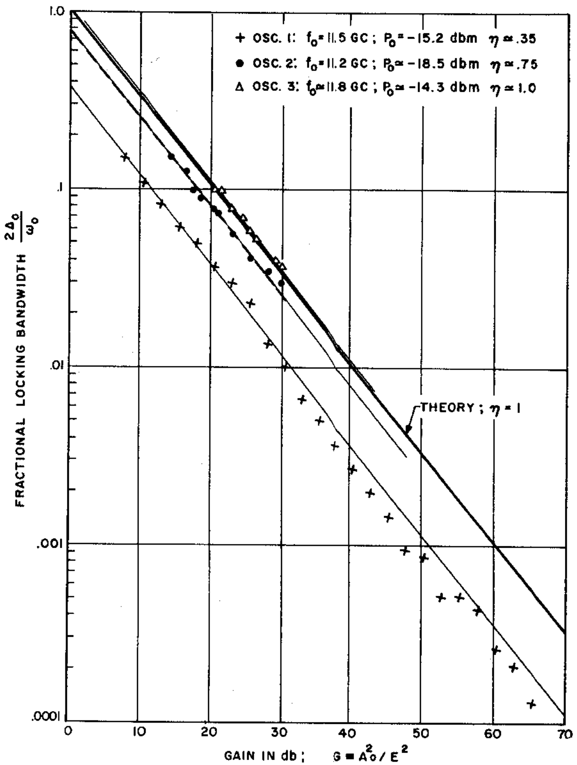

FIG. 3 VERIFICATION OF THE EXPRESSION: $\frac{2\Delta_0}{\omega_0} = \eta \frac{E_0}{A_0}$
FRACTIONAL LOCKING BANDWIDTH VS. POWER GAIN.
VERTICAL INTERCEPT IS η.

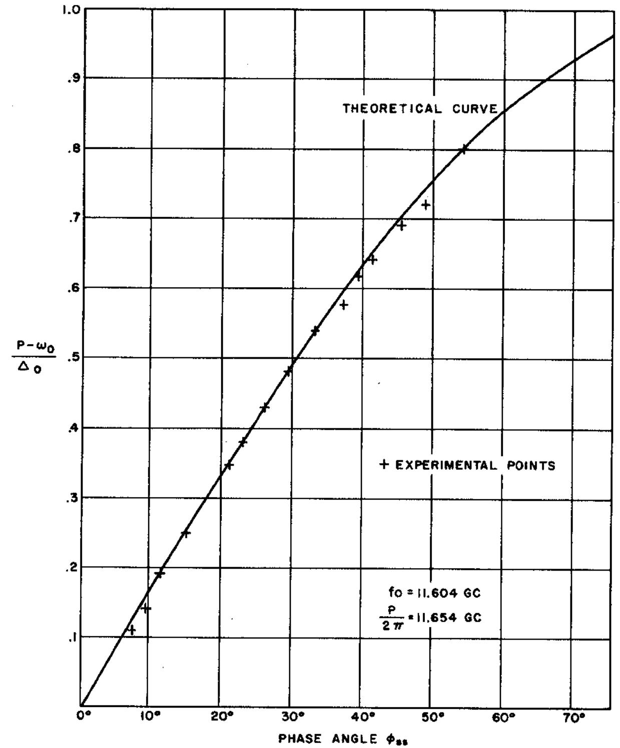

FIGURE 4
VERIFICATION OF THE EXPRESSION: $\sin \phi_{ss} = \frac{P-\omega_0}{\Delta_0}$

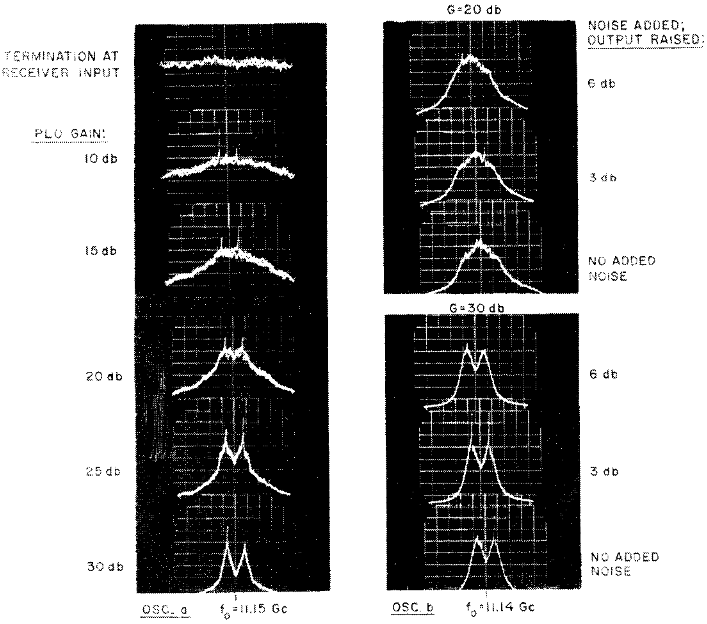

FIGURE 5
OUTPUT NOISE SPECTRA OF PHASE-LOCKED
TUNNEL DIODE OSCILLATOR.
FREQUENCY SCALE=100 MCS/DIV.

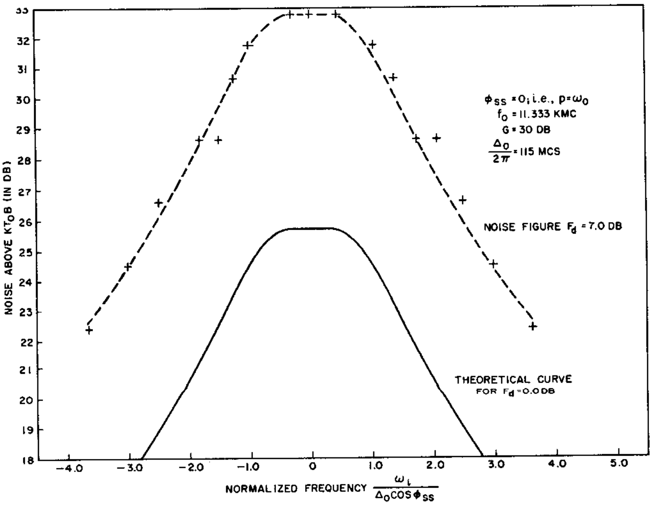

FIGURE 6–OUTPUT NOISE SPECTRUM, THEORY AND EXPERIMENT
(NOTE: THE I.F. FREQUENCY OF THE DOUBLE SIDEBAND RECEIVER IS 70 MCS.)